2. Let X_1 denote the student's score on test A and let X_2 denote his score on test B. The conditional distribution of X_2 given that $X_1 = 80$ is a normal distribution with mean $90 + (0.8)(16)\left(\frac{80-85}{10}\right) = 83.6$ and variance $(1 - 0.64)(256) = 92.16$. Therefore, given that $X_1 = 80$, the random variable $Z = (X_2 - 83.6)/9.6$ will have a standard normal distribution. It follows that

$$\Pr(X_2 > 90 \,|\, X_1 = 80) = \Pr\left(Z > \frac{2}{3}\right) = 1 - \Phi\left(\frac{2}{3}\right) = 0.2524.$$

6. $\mathrm{Var}(X_1 + bX_2) = \sigma_1^2 + b^2\sigma_2^2 + 2b\rho\sigma_1\sigma_2$. This is a quadratic function of b. By differentiating with respect to b and setting the derivative equal to 0, we obtain the value $b = -\rho\sigma_1/\sigma_2$.

14. The marginal p.d.f. of X is

$$f_1(x) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{1}{2\sigma^2}[x-\mu]^2\right),$$

where μ and σ^2 are the mean and variance of X. The conditional p.d.f. of Y given $X = x$ is

$$g_2(y|x) = \frac{1}{(2\pi\tau^2)^{1/2}} \exp\left(-\frac{1}{2\tau^2}[y-ax-b]^2\right).$$

The joint p.d.f. of (X, Y) is the product of these two

$$f(x,y) = \frac{1}{2\pi\sigma\tau} \exp\left(-[a'x^2 + b'y^2 + cxy + ex + gy + h]\right),$$

where

$$\begin{aligned}
a' &= \frac{1}{2\sigma^2} + \frac{a^2}{2\tau^2}, \\
b' &= \frac{1}{2\tau^2}, \\
c &= -\frac{a}{\tau^2}, \\
e &= -\frac{\mu}{\sigma^2} + \frac{ab}{\tau^2}, \\
g &= -\frac{b}{\tau^2},
\end{aligned}$$

and h is irrelevant since we are going to apply the result from Exercise 13. Clearly a' and b' are positive. We only need to check that $a'b' > (c/2)^2$. Notice that

$$a'b' = \frac{1}{4\sigma^2\tau^2} + \frac{a^2}{4\tau^4} = (c/2)^2 + \frac{1}{4\sigma^2\tau^2},$$

so the conditions of Exercise 13 are met.